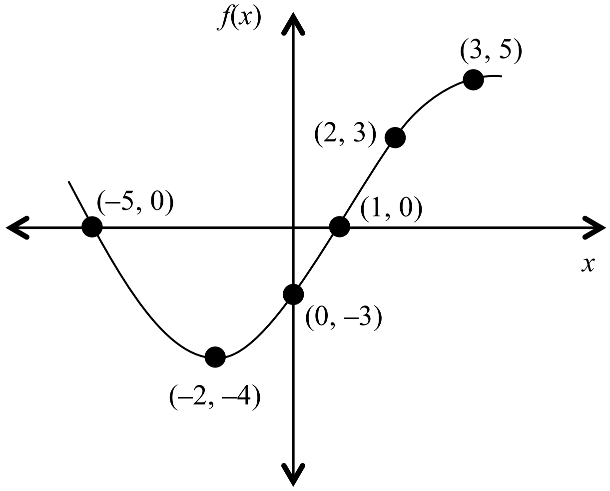
Grade Level/Course: Algebra 1
Lesson/Unit Plan Name: Comparing Linear and Quadratic Functions
Rationale/Lesson Abstract: This lesson will enable students to compare the properties of linear and quadratic functions each expressed in a different way.
Timeframe: 50 minutes + up to 60 minute assessment/extension activity
Common Core Standard(s): F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Instructional Resources/Materials: Copies of student notes, copies of the warm-up, copies of function cards for each student or pair of students, pencil.

Warm-Up

A portion of the graph of $f(x)$ is below. Indicate which of the following is true for $f(x)$.

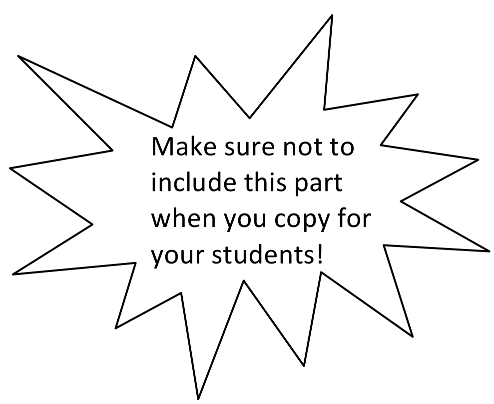


A.	$f(x)$ is increasing on the interval $0 \leq x \leq 3$	○ True	○ False
B.	$f(x)$ is increasing everywhere	○ True	○ False
C.	$f(x)$ has at least two x-intercepts	○ True	○ False
D.	$-4 < f(-3) < 0$	○ True	○ False
E.	$f(0) = 1$	○ True	○ False
F.	The y-coordinate of the y-intercept of $f(x)$ is -3.	○ True	○ False

F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

WARM-UP Answer Key:

A: True
B: False
C: True
D: True
E: False
F: True

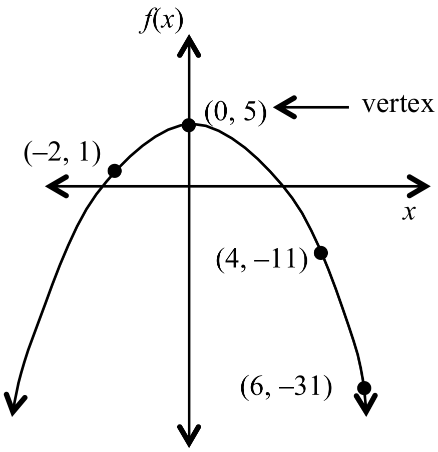


Activity/Lesson:

Part 1: Identify key features from various representations

Example 1: "I do"

Identify the following key features of the quadratic function that is shown on the graph below:



A. The average rate of change from $-2 \le x \le 6$

• **What part of the graph are we looking at?** The section between – 2 and 6 on the *x*-axis. **Highlight with pencil or whiteboard marker this section of the graph**

• **Can we tell from the graph if it's increasing or decreasing on that interval?** From –2 to 0 it's increasing and from 0 to 6 it's decreasing. It's decreasing more than increasing, so we can assume that the average rate of change will be negative.

• **How do we find the average rate of change?** We can use the following formula:

$$\frac{f(b)-f(a)}{b-a} \text{ for } b=6 \text{ and } a=-2$$

$$\frac{f(6)-f(-2)}{6-(-2)}$$

$$=\frac{-31-1}{6+2}$$

At *x* = 6, the *y*-value of the graph is –31.
At *x* = –2, the *y*-value of the graph is 1.

$$=\frac{-32}{8}$$

$$=-4$$

∴ The average rate of change on that interval is – 4.

B. The maximum value of the function over the interval $-2 \le x \le 6$

• **What part of the graph are we looking at?** The section between –2 and 6 on the *x*-axis. **Highlight with pencil or whiteboard marker this section of the graph**

• **Are we looking for the highest/lowest/left-most/right-most point?** The highest point (because it is asking for the maximum of the function and the function values are graphed on the *y*-axis).

• **What is the highest point on that interval?** The vertex; (0, 5). Therefore the maximum on that interval is 5.

C. The *y*-coordinate of the *y*-intercept.

• **Where can we find the *y*-intercept?** Where the graph crosses the *y*-axis. **Highlight this point**

• **What is the *y*-value/coordinate at this point?** $y = 5$

D. The approximation for $f(-1)$

• **Point to the place on the graph where we can find *f*(–1).** **Highlight this point**

• **Is this value positive or negative?** Positive.

• **Which numbers will it be in between?** Between 1 and 5 because $x = -1$ is in between $x = -2$ and $x = 0$ **Highlight these function values on the graph. Show that the point on the graph is at a height/*y*-value somewhere between 1 and 5**

• **What are some appropriate estimates for the value of *f*(–1)?** 4, 3, or even 2. **We are only looking for estimations, but you can point out that the height is closer to 5 than it would be to 1; therefore 4 or 3 are more accurate estimations than 2**

You Try #1:

Identify the following key features of the linear function that created this table:

x	$g(x)$
–2	–3
0	–2
2	–1
4	0
6	1

A. The average rate of change from –2 ≤ *x* ≤ 2

- **What part of the table are we looking at?** The first three entries. **Circle that section of the table**
- **Are the function values increasing or decreasing on that interval?** Increasing, going from MORE negative to LESS negative. **Show on the number line to aid understanding**
- **Do we expect our average rate of change to be positive?** Yes.
- **How do we find the average rate of change?** Use the following formula:

$$\frac{f(b)-f(a)}{b-a} \text{ for } b=2 \text{ and } a=-2$$

$$\frac{g(2)-g(-2)}{2-(-2)}$$

$$=\frac{-1-(-3)}{2+2}$$

According to the table, *g*(2) = –1 and *g*(–2) = –3.

$$=\frac{2}{4}$$

$$=\frac{1}{2}$$

The average rate of change on that interval is $\frac{1}{2}$.

- **If we chose any interval, would we get the same rate of change?** Yes. **Why?** The function is linear. (It tells us this in the description/directions.)

B. The minimum value of the function over the interval –2 ≤ *x* ≤ 2

- **Which part of the table are we looking at?** The first three entries. **Circle that section of the table**
- **Are we looking for the highest or lowest value?** Lowest; because it's asking for the minimum.
- **Are we looking at the values of *x* or *g*(*x*)?** *g*(*x*); because we are looking for the values of the function.
- **What is the lowest value for *g*(*x*) on that interval?** –3 **Circle that value on the table**

C. The *y*-coordinate of the *y*-intercept.

- **How do we find the *y*-intercept on a table?** It is the value of the function when *x* = 0. **Why?** Because any point where *x* = 0 is on the *y*-axis. **Circle that point in the table**
- **What is the *y*-value/coordinate when *x* = 0?** –2.

D. *g*(–1)

- **Is *g*(–1) going to be positive or negative?** It will be negative because it is in between two negative values on the table. **Draw an arrow to the space between table entries where *g*(–1) would be**
- **Which values will it be in between?** Between –3 and –2.
- **Can we determine an exact value? Why/why not?** Yes. We know that *g*(*x*) is linear so the rate of change is constant. That means for every unit of *x*, *g*(*x*) is increasing by $\frac{1}{2}$. This means:

$$g(-1)=g(-2)+\frac{1}{2}$$

$$g(-1)=-3+\frac{1}{2}$$

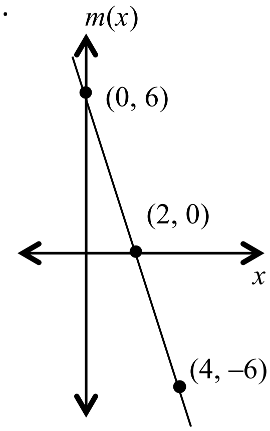
$$g(-1)=-2\frac{1}{2}=-2.5$$

Part 2: Comparing two functions using their properties

Example 2: "I do"

The algebraic representation of a quadratic function $n(x)$ is shown below. A portion of the graph of a linear function $m(x)$ is shown in the xy-plane.

$$n(x) = x^2 - 5x + 6$$



For each comparison below, select the correct symbol (< , >, or =) that indicates the relationship between the first and second quantity.

	First Quantity	Comparison (<, >, or =)	Second Quantity	
6	y-coordinate of y-intercept of $n(x)$	=	y-coordinate of y-intercept of $m(x)$	6
–3	average rate of change of $n(x)$ on the interval $0 \le x \le 2$	=	average rate of change of $m(x)$ on the interval $0 \le x \le 2$	–3
20	maximum of $n(x)$ on the interval $-2 \le x \le 0$	>	maximum of $m(x)$ on the interval $-2 \le x \le 0$	12
2	$n(1)$	<	$m(1)$	3

Y-Intercepts:

- $n(x)$: **To find the y-intercept of $n(x)$ you can graph it or evaluate the function when $x = 0$. I will choose to evaluate it:**

$$n(x) = x^2 - 5x + 6$$
$$n(0) = (0)^2 - 5(0) + 6$$
$$n(0) = 0 - 0 + 6$$
$$n(0) = 6$$

Therefore, the y-coordinate of the y-intercept of $n(x)$ is 6.

- $m(x)$: **Where on the graph is the y-intercept?** Where it crosses the y-axis. **Highlight the point**
  What is the y-value/coordinate at this point? 6

These values are equal!

Average Rate of Change:

• $n(x)$: **Is this parabola concave up or concave down?** Concave up; the coefficient on x^2 is positive.
Can we tell if it's increasing or decreasing on this interval? No. It could be either.
How do we find the average rate of change over an interval?

$$\frac{f(b)-f(a)}{b-a} \text{ for } b=2 \text{ and } a=0$$

$$\frac{n(2)-n(0)}{2-0} = \frac{0-6}{2-0} = \frac{-6}{2} = -3$$

Side work for finding *n*(2):

$$n(2)=(2)^2-5(2)+6$$
$$n(2)=4-10+6$$
$$n(2)=0$$

We know *n*(0) = 6 from earlier.

∴ The average rate of change on that interval is –3.

• $m(x)$: **Is this graph increasing or decreasing on that interval?** Decreasing (on all intervals).
How do we find the average rate of change over an interval?

$$\frac{f(b)-f(a)}{b-a} \text{ for } b=2 \text{ and } a=0$$

$$\frac{m(2)-m(0)}{2-0} = \frac{0-6}{2-0} = \frac{-6}{2} = -3$$

At $x = 2$, the *y*-value of the graph is 0.
At $x = 0$, the *y*-value of the graph is 6.

∴ The average rate of change on that interval is –3.

These values are equal!

Maximums:

• $n(x)$: **Do we know if the function is increasing or decreasing on that interval?** We know that it is decreasing on $0 \le x \le 2$. $x=\frac{-b}{2a}=\frac{5}{2}$ is the vertex of this parabola, so we know everywhere to the left of that will be decreasing. Therefore, the function is decreasing on this interval.
Since it is decreasing, where will the maximum be? At the smallest *x*-value on that interval.
Find *n*(–2). $n(-2) = (-2)^2 - 5(-2) + 6$
$= 4 + 10 + 6$
$= 20$

• $m(x)$: **Do we know if the function is increasing or decreasing on that interval?** It is decreasing.
Since it is decreasing where will the maximum be? At the smallest *x*-value on that interval.
Find *m*(–2). Since the function is decreasing constantly at a rate of –3, we can find $m(-2)$ by increasing at a rate of 3 from the last known point. $m(-2) = m(0) + 3 + 3$
$= 6 + 3 + 3$
$= 12$

The first quantity is greater!

Evaluating:

• $n(x)$: **We can evaluate the function at this point.**

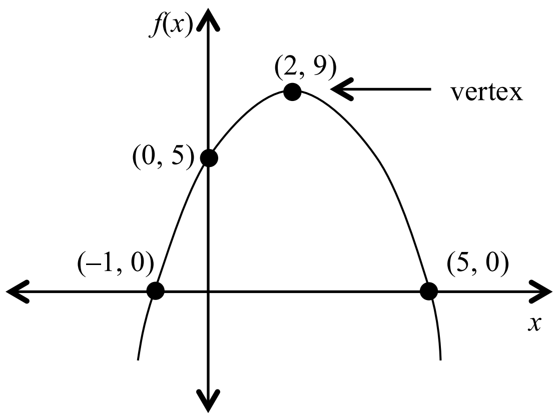
$$n(1)=(1)^2-5(1)+6$$
$$=1-5+6$$
$$=2$$

• $m(x)$: **Where should this point be on the graph?** **Point to it**
What values is it between? Between 6 and 0
Can we find the exact value? Yes. It is decreasing at a rate of –3 ∴ $m(1) = m(0) - 3 = 6 - 3 = 3$

The second quantity is greater!

You Try #2:

A portion of the graph of a quadratic function *f*(*x*) is shown in the *xy*-plane. Selected values of a linear function *g*(*x*) are shown in the table.



x	*g*(*x*)
–4	7
–1	1
2	–5
5	–11

For each comparison below, select the correct symbol (< , >, or =) that indicates the relationship between the first and second quantity.

	First Quantity	Comparison (<, >, or =)	Second Quantity	
5	*y*-coordinate of *y*-intercept of *f*(*x*)	>	*y*-coordinate of *y*-intercept of *g*(*x*)	between 1 and –5
Positive	*f*(3)	>	*g*(3)	Negative
–3	average rate of change of *f*(*x*) on the interval $2 \le x \le 5$	<	average rate of change of *g*(*x*) on the interval $2 \le x \le 5$	–2
9	maximum of *f*(*x*) on the interval $-5 \le x \le 5$	=	maximum of *g*(*x*) on the interval $-5 \le x \le 5$	9

Y-Intercept:

- *f*(*x*): **Do we have the exact value?** Yes. **Where is it?** Where the function crosses the *y*-axis. 5.
- *g*(*x*): **Do we have the exact value?** No. **Where would it be?** At $x = 0$. **Indicate on table**
  Can we approximate? Yes. *g*(0) will be somewhere between 1 and –5.
  Is this enough information to make our comparison? Yes; we know it will be smaller than 1.

The first quantity is greater!

Evaluate:

- *f*(*x*): **Is it labeled on the graph?** No.
  Find where *f*(3) would be on the graph. **Point to it** **Will it be positive or negative?** Positive.
- *g*(*x*): **Is it on the table?** No.
  Find where *g*(3) would be on the table. **Point to it** **Will it be positive or negative?** Negative

***Point out to students that, by asking only these questions, we have enough information to compare the two quantities. However, if they would like to approximate further, they can use the methods outlined in the first part of the lesson to approximate that $f(3) \approx 8$, or 7 and $g(3) \approx -7$.

The first quantity is greater!

Average Rate of Change:

- $f(x)$: **Which portion of the graph are we looking at?** The part between $x = 2$ and $x = 5$.
  Is the graph increasing or decreasing on that interval? Decreasing.
  How do we find the average rate of change? By using the following formula:

$$\frac{f(b)-f(a)}{b-a} \text{ for } b=5 \text{ and } a=2$$

$$\frac{f(5)-f(2)}{5-2}$$

$$=\frac{0-9}{5-2}$$

At $x = 5$, the y-value of the graph is 0.
At $x = 2$, the y-value of the graph is 9.

$$=\frac{-9}{3}$$

$$=-3$$

∴ The average rate of change on that interval is –3.

- $g(x)$: **Which portion of the table are we looking at?** The last two entries. **Highlight those points**
  Are the values of $g(x)$ increasing or decreasing over those points? Decreasing.
  How do we find the average rate of change? By using the following formula:

$$\frac{f(b)-f(a)}{b-a} \text{ for } b=5 \text{ and } a=2$$

$$\frac{g(5)-g(2)}{5-2}$$

$$=\frac{-11-(-5)}{5-2}$$

According to the table, $g(5) = -11$ and $g(2) = -5$.

$$=\frac{-6}{3}$$

$$=-2$$

∴ The average rate of change on that interval is –2.

The second quantity is greater!

Maximums:

- $f(x)$: **What part of the graph are we looking at?** The part between $x = -5$ and $x = 5$ **Highlight this part of the graph**
  Are we looking for the highest/lowest/left-most/right-most point? The highest point (because it is asking for the maximum of the function and the function values are on the y-axis).
  What is the highest point on that interval? The vertex: (2, 9). Therefore the maximum on that interval is 9.
- $g(x)$: **Is this function increasing or decreasing?** It is decreasing at a rate of –2. It is decreasing EVERYWHERE at this constant rate because it is linear.
  Where will we find the maximum value? At the minimum x-value because it is decreasing.
  What is $g(-5)$? Because $g(x)$ is linear and decreasing at a constant rate of –2, we can find $g(-5)$ by increasing at a rate of 2 from $g(-4)$.

$g(-5) = g(-4) + 2$
$g(-5) = 7 + 2$
$g(-5) = 9$

The quantities are equal!

Assessment/Extension:

The assessment/extension activity can be used in the way that's the most productive for your students. You are provided with a set of 6 cards, each with a different function, represented in a different way (equation, graph, or table). If you see that your students are struggling with finding certain key features of functions, start by asking them to order the cards from least to greatest as determined by that key feature. If the feature is equal on two or more functions, place them next to each other.

You may choose from the following completed orderings, or create your own.

<u>Ordering Options:</u>

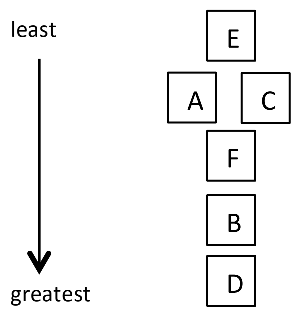
Directions: Order the cards from least to greatest. If quantities are equal, line them up next to each other.

1. Maximum on the interval $2 \le x \le 6$

Correct values:

A: –1
B: 20.25
C: –1
D: 32
E: –4
F: 1

Students should order the cards like this:

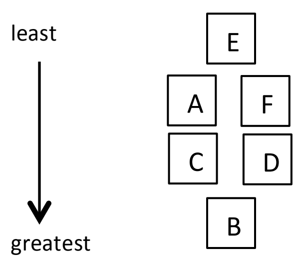


2. Minimum on the interval $0 \le x \le 4$

Correct values:

A: –5
B: 8
C: –4
D: –4
E: –8
F: –5

Students should order the cards like this:

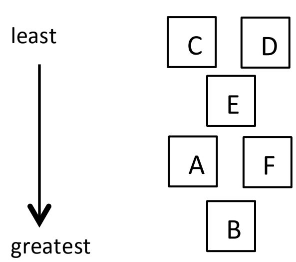


3. *y*-coordinate of the *y*-intercept

Correct values:

A: 3
B: 8
C: –4
D: –4
E: 0
F: 3

Students should order the cards like this:

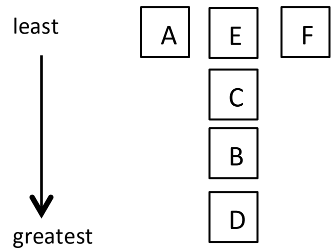


4. Average rate of change on the interval $0 \le x \le 4$

Correct values:

A: –2
B: 3
C: $\frac{1}{2}$
D: 4
E: –2
F: –2

Students should order the cards like this:

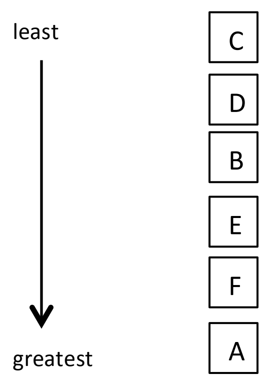


5. Functions evaluated at $x = -1$

Correct values:

A: 5
B: 0
C: $-5 < c(-1) < -4$
D: –3
E: 2
F: $\frac{5}{2}$

Students should order the cards like this:



Card Sort: Cut out and give a set to each student/pair of students.

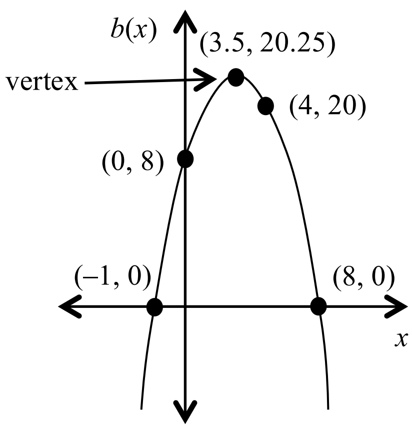
A

The algebraic representation of the linear function *a*(*x*) is shown below.

$$a(x) = -2x + 3$$

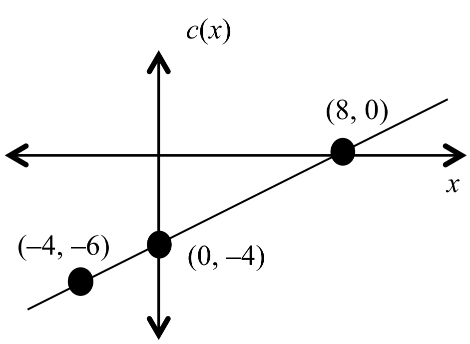
B

A portion of the graph of the quadratic function *b*(*x*) is shown below.



C

A portion of the graph of the linear function *c*(*x*) is shown below.



D

The algebraic representation of the quadratic function *d*(*x*) is shown below.

$$d(x) = x^2 - 4$$

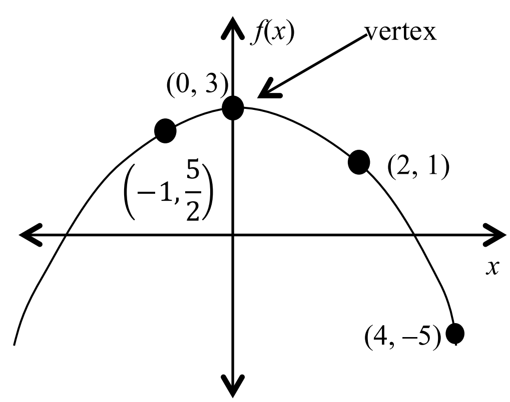
E

Selected values of a linear function *e*(*x*) are shown in the table below.

x	$e(x)$
–3	6
–1	2
1	–2
3	–6
5	–10

F

A portion of the graph of the quadratic function *f*(*x*) is shown below.

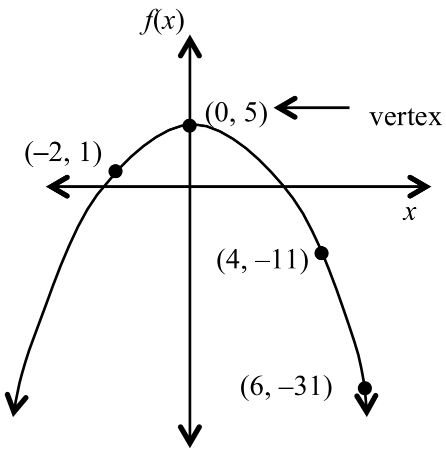


STUDENT NOTES:

Part 1: Identify key features from various representations

Example 1: "I do"

Identify the following key features of the quadratic function that is shown on the graph below:



A. The average rate of change from $-2 \le x \le 6$

B. The maximum value of the function over the interval $-2 \le x \le 6$

C. The *y*-coordinate of the *y*-intercept.

D. The approximation for $f(-1)$

You Try #1:

Identify the following key features of the linear function that created this table:

x	$g(x)$
–2	–3
0	–2
2	–1
4	0
6	1

A. The average rate of change from $-2 \le x \le 2$

B. The minimum value of the function over the interval $-2 \le x \le 2$

C. The y-coordinate of the y-intercept.

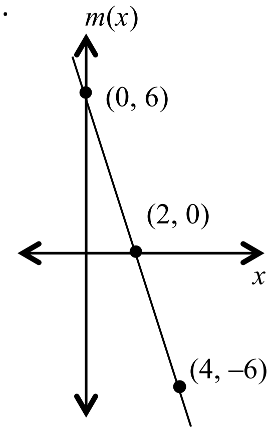
D. $g(-1)$

Part 2: Comparing two functions using their properties

Example 2: "I do"

The algebraic representation of a quadratic function $n(x)$ is shown below. A portion of the graph of a linear function $m(x)$ is shown in the xy-plane.

$$n(x) = x^2 - 5x + 6$$

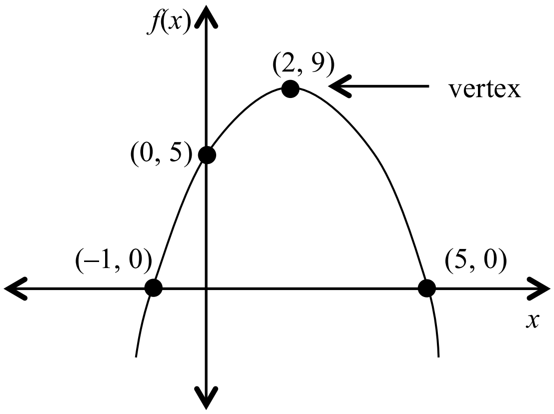


For each comparison below, select the correct symbol (< , >, or =) that indicates the relationship between the first and second quantity.

First Quantity	**Comparison (<, >, or =)**	**Second Quantity**
y-coordinate of y-intercept of $n(x)$		y-coordinate of y-intercept of $m(x)$
average rate of change of $n(x)$ on the interval $0 \le x \le 2$		average rate of change of $m(x)$ on the interval $0 \le x \le 2$
maximum of $n(x)$ on the interval $-2 \le x \le 0$		maximum of $m(x)$ on the interval $-2 \le x \le 0$
$n(1)$		$m(1)$

You Try #2:

A portion of the graph of a quadratic function *f*(*x*) is shown in the *xy*-plane. Selected values of a linear function *g*(*x*) are shown in the table.



x	**g(x)**
–4	7
–1	1
2	–5
5	–11

For each comparison below, select the correct symbol (< , >, or =) that indicates the relationship between the first and second quantity.

First Quantity	**Comparison (<, >, or =)**	**Second Quantity**
y-coordinate of *y*-intercept of *f*(*x*)		*y*-coordinate of *y*-intercept of *g*(*x*)
f(3)		*g*(3)
average rate of change of *f*(*x*) on the interval $2 \le x \le 5$		average rate of change of *g*(*x*) on the interval $2 \le x \le 5$
maximum of *f*(*x*) on the interval $-5 \le x \le 5$		maximum of *g*(*x*) on the interval $-5 \le x \le 5$